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Topological structure of the SU(3) vacuum and exceptional eigenmodes of the improved Wilson-Dirac operator*

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We present a study of the instanton size and spatial distributions in pure SU(3) gauge theory using under-relaxed cooling. We also investigate the low-lying eigenmodes of the (improved) Wilson-Dirac operator, in particular, the appearance of zero-modes and their space-time localisation with respect to instantons in the underlying gauge field.

Instanton content of the SU(3) vacuum [1]

The importance of the instanton content of $SU(3)$ gauge theory comes through both the intrinsic importance of understanding the ground state of QCD and the role instantons are conjectured to play in light hadron structure. Cooling is a technique for removing the high-frequency non-topological excitations of the gauge-field. However, during cooling instantons are also removed; either if they are very small (lattice artifacts) or through $I\bar{I}$ annihilation. We use under-relaxed cooling to reduce the latter problem. Also, on the cooled configurations there is still the problem of extracting the instanton properties; for this we have developed pattern-recognition algorithms. We present results for 20 configurations at $\beta = 6.0$ ($16^3 48$) and $\beta = 6.2$ ($24^3 48$) lattices.

The gauge update for under-relaxed cooling [3] is implemented in each Cabibbo-Marinari subgroup as

$$U_{new} = c(U_{min} + \alpha U_{old}) \qquad (1)$$

where U_{min} is the gauge link that minimises the action, U_{old} the original link, α is the under-relaxation parameter and c a normalisation constant. Under-relaxed cooling increases the number of *calibrated* sweeps needed to annihilate an $I\bar{I}$ pair; for a given value of α a calibrated sweep is the number of sweeps needed to destroy a $\rho = 2$ instanton. With no under-relaxation one occasionally finds a very narrow instanton broadening out under cooling (presumably because of its environment). We have not observed this with (significant) under-relaxation. We chose $\alpha = 1$ and our measurements were carried out between 23 and 46 cooling sweeps (corresponding to between 10 and 20 cooling sweeps at $\alpha = 0$).

On the cooled configurations we first find all the local extrema of the symmetrised topological charge density, $Q(x)$, relative to the 3^4 block surrounding each point. (We do not consider the action, $S(x)$, as it clearly records less structure.) Each peak is treated as a linear superposition of the topological charge of the object at that point, calculated from a lattice-corrected formula, plus a contribution from every other object on the lattice, calculated from the continuum formula. A self-consistent set of widths is then found by iteration. These are our candidate instantons.

Summing up $Q(x)$ over the lattice and comparing it to $n_I - n_{\bar{I}}$ shows a discrepancy. We define

$$\delta = < |Q - (n_I - n_{\bar{I}})| > \qquad (2)$$

and impose filters on our candidate instantons. The parameters of the filters are chosen to minimise δ. We have a "spatial" filter to remove spurious peaks due to ripples on large objects and a "width" filter. The latter compares the width cal-

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culated above with the width calculated from the charge within a radius 2 (or 3) of the peak using a lattice-corrected formula; a peak is only included if the various widths are in sufficiently good agreement. Full details will appear elsewhere [1].

In Figure 1 we show the instanton size distribution for $\beta = 6.2$ at 23 sweeps. The distribution

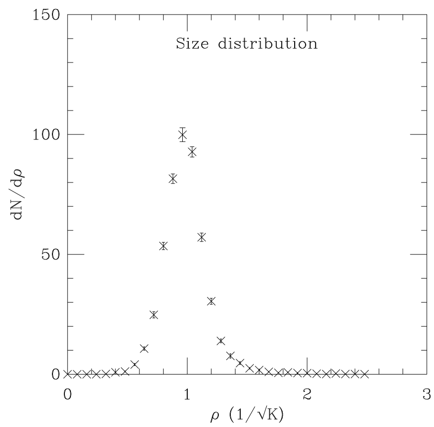

Figure 1. Size distribution: $\beta = 6.2$, 23 sweeps

is peaked around $\rho \approx \frac{1}{\sqrt{K}}$. The best fit to the large-ρ tail of the distribution is $D(\rho) \propto \rho^{-\alpha}$ with $\alpha \approx 10$. As one would expect, the total number of instantons is found to vary rapidly with the amount of cooling. However the average size and the form of the small/large ρ tails varies much less. We note that our results are consistent with those of [4] but not with those of [5].

The fact that the impact of a cooling sweep does not scale complicates the scaling analysis. Figure 2 shows that we can tune the number of cooling sweeps so as to get scaling when comparing $\beta = 6.2$ and $\beta = 6.0$.

Examining the number of unlike charges a distance R away from each peak (normalised by the volume of the shell) gives the distribution that is shown in Fig 3 (for $\beta = 6.2$ after 23 sweeps). It is uniform at long distances and amplified at short distances. The corresponding distribution for like charges is uniform at long distances and suppressed at short distances. This implies some

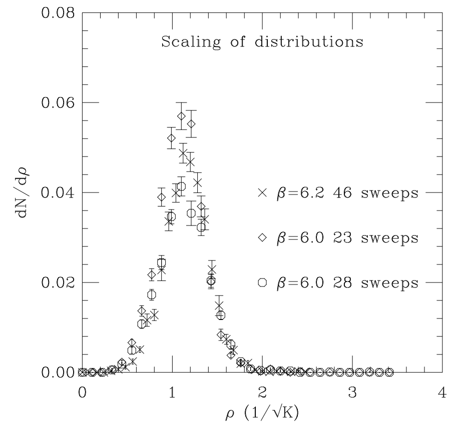

Figure 2. Scaling of size distributions

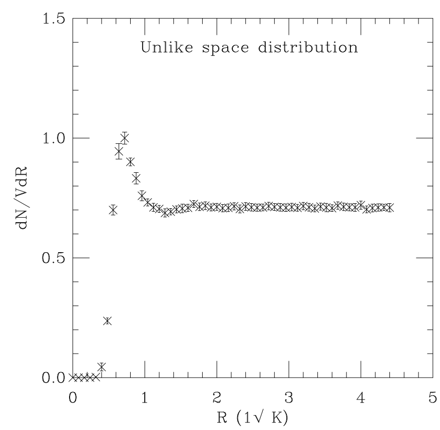

Figure 3. Unlike Spatial Distribution

screening of instantons by anti-instantons in the vacuum – as expected.

Calculating $< \frac{Q}{|Q|} \frac{q(\rho)}{n(\rho)} >$ where $n(\rho)$ is the number of objects of size ρ and $q(\rho)$ is the charge carried by objects of size ρ shows that the charge carried by small (large) instantons is correlated (anti-correlated) with the sign of Q (Figure 4). Indeed our results suggest over-screening of large instantons by small anti-instantons.

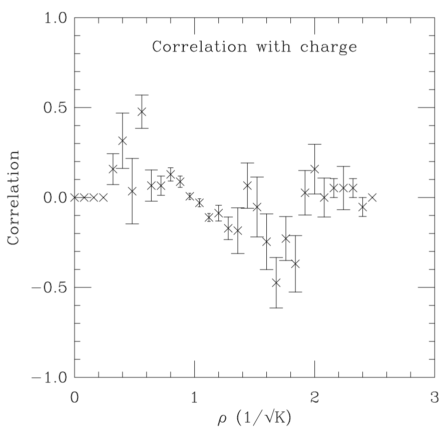

Figure 4. $< \frac{Q}{|Q|} \cdot \frac{q(\rho)}{n(\rho)} >$, $\beta = 6.2$, 23 sweeps

Zero modes of the improved Wilson-Dirac Operator [2]

The Wilson-Dirac operator, or equivalently $\mathbf{Q}(\kappa) = \gamma_5 M$, may have vanishing or almost-zero eigenvalues for certain values of κ. Moreover, it has been shown that the lowest eigenvalues of $\mathbf{Q}$ are strongly localised in space-time [6].

"Exceptional" configurations are assumed to be related to the appearance of (almost-)zero eigenvalues at some $\kappa_0 < \kappa_{crit}$. They seem to be more frequent at smaller β and with SW improvement. On the other hand, it is unclear if and how these zero modes are related to the topology of the underlying gauge field, in particular because the chirality $\chi = \langle \psi | \gamma_5 | \psi \rangle$ of the corresponding eigenvectors ψ of $\mathbf{Q}(\kappa_0)$ is typically much smaller than one. To clarify this relation we investigate the low-lying eigenmodes of $\mathbf{Q}(\kappa)$ using a modified conjugate gradient method [7]. We verified that the index theorem is realized on single instanton configurations (generated as in ref. [8]), for both improved and unimproved Wilson Fermions, with one right-handed (4 right-handed plus 3 left-handed) zero modes in the range $\kappa < 0.2$ for anti-periodic (periodic) boundary conditions. This is in complete analogy to the results for 2-dimensional Wilson and for staggered fermions [8].

For anti-periodic boundary conditions, the eigenmodes are localised and centered on the instanton. Moreover, we find considerable effects from discretization errors and SW improvement which are summarized in table 1.

Table 1
Position κ_0 and chirality χ of the zero-mode for single instantons without (with) improvement.

ρ	c_{sw}	κ_0	χ
2	0 (1)	0.135 (0.126)	0.66 (0.986)
3	0 (1)	0.129 (0.125)	0.79 (0.999)
4	0 (1)	0.127 (0.125)	0.90 (0.999)

We also investigated the localisation of the lowest eigenmodes on four exceptional configurations encountered by UKQCD at $\beta = 6.0$ on $16^3 48$ and $32^3 64$ lattices. In all cases the (almost-)zero mode is localized close to a small ($\rho = 2a \dots 3a$) instanton (less than $\sqrt{2}a$ away). The chirality has the same sign as the topological charge of the topological object but is not correlated with the overall topological charge of the configuration. This provides some evidence that exceptional configurations are related to small instantons in the underlying gauge field. More detailed results will be presented elsewhere [2].

1. Acknowledgements

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